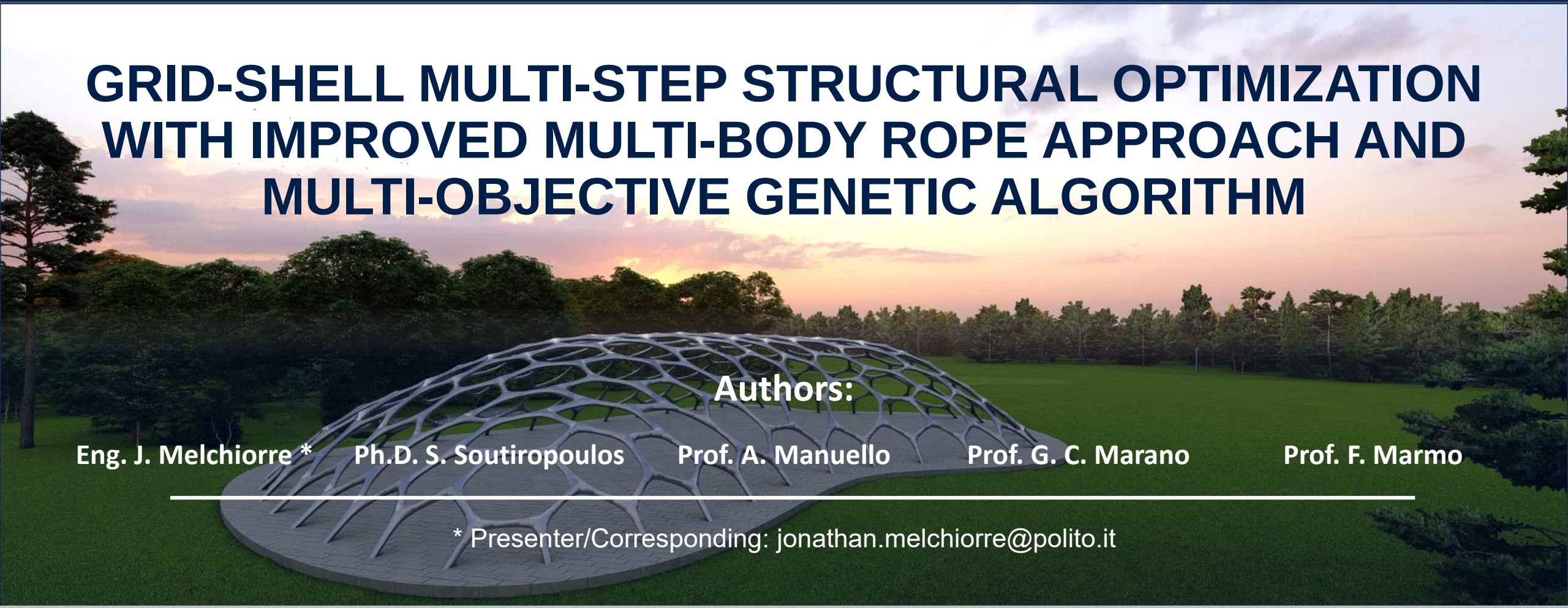
IWSS 2023
The Italian Workshop on Shell and Spatial Structure

# GRID-SHELL MULTI-STEP STRUCTURAL OPTIMIZATION WITH IMPROVED MULTI-BODY ROPE APPROACH AND MULTI-OBJECTIVE GENETIC ALGORITHM



**Authors:**

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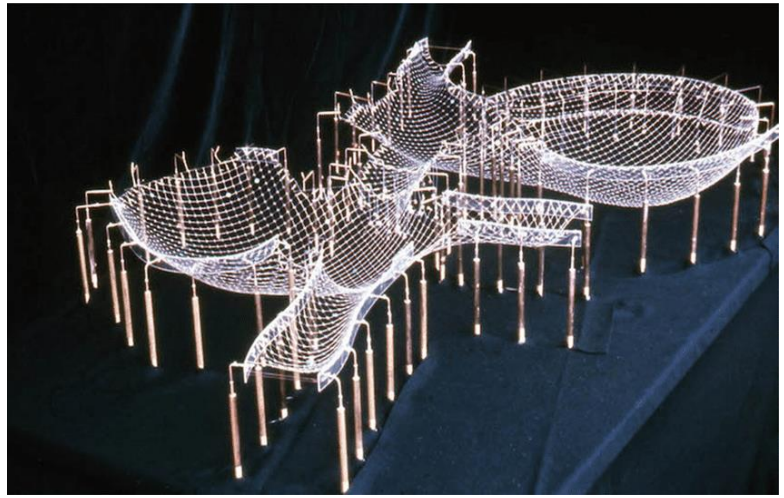
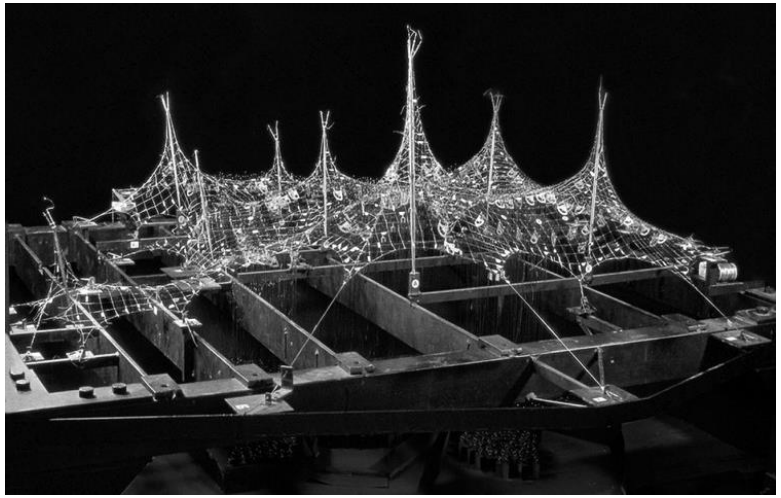
# Gridshell and form-finding methods

**Gridshell** roofing constructions are popular for their ability to create large, **lightweight roofs** with **slender main structural elements**. Gridshell structures are defined by the **interaction between their shape and stress distribution** thus, **designing** such structures **directly**, as in the case of conventional structures, **is ineffective**.





In the 1960s and 70s innovative **new methods for gridshell design** were carried out.

**Form-finding techniques**

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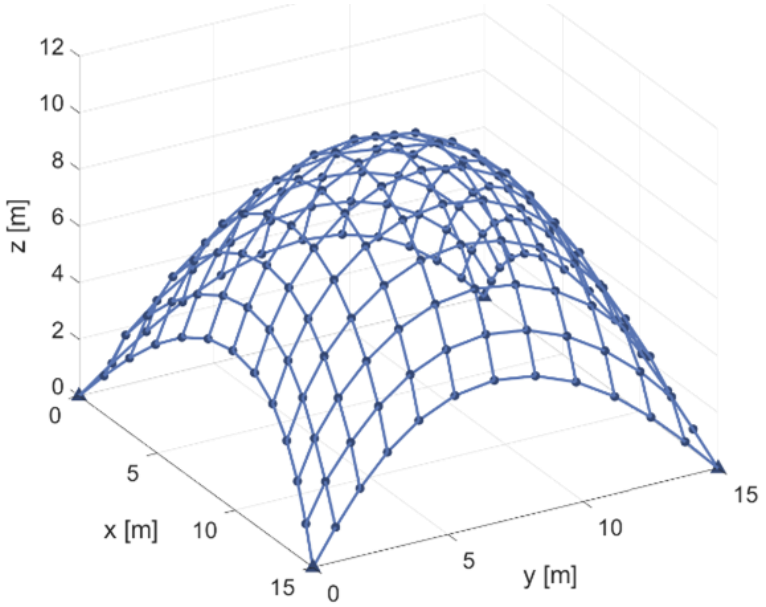
# Multibody Rope Approach (MRA)

The **MRA** is an original method specifically designed for determining the form of **gridshell structures** that use **free-forms** and **standardized building elements**.




MRA utilizes a **dynamic model of falling bodies** to model the hanging network (**funicular configuration**) using **ropes**, thus means that no force is exerted if the distance between the ends is less than the rope's fixed length.

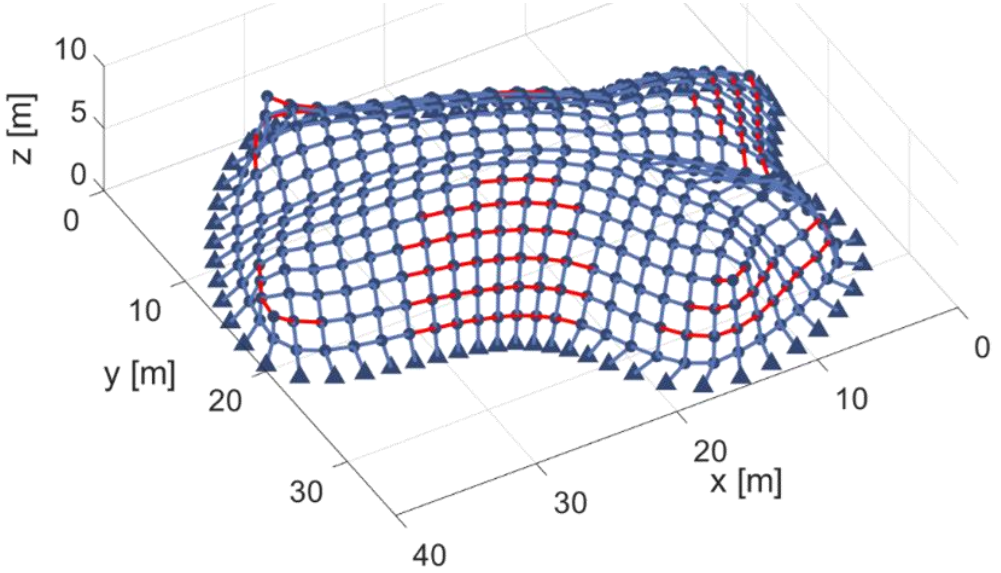
$$\begin{cases} F_{rope} = 0 & if \quad l < l_{rope} \\ F_{rope} = k(l - l_{rope}) & if \quad l \geq l_{rope} \end{cases}$$

# Improved Multibody Rope Approach (i-MRA)

The **i-MRA** method improves on MRA by integrating techniques that **optimize the structural geometry** for both **structural reasons and automation of the construction process**.

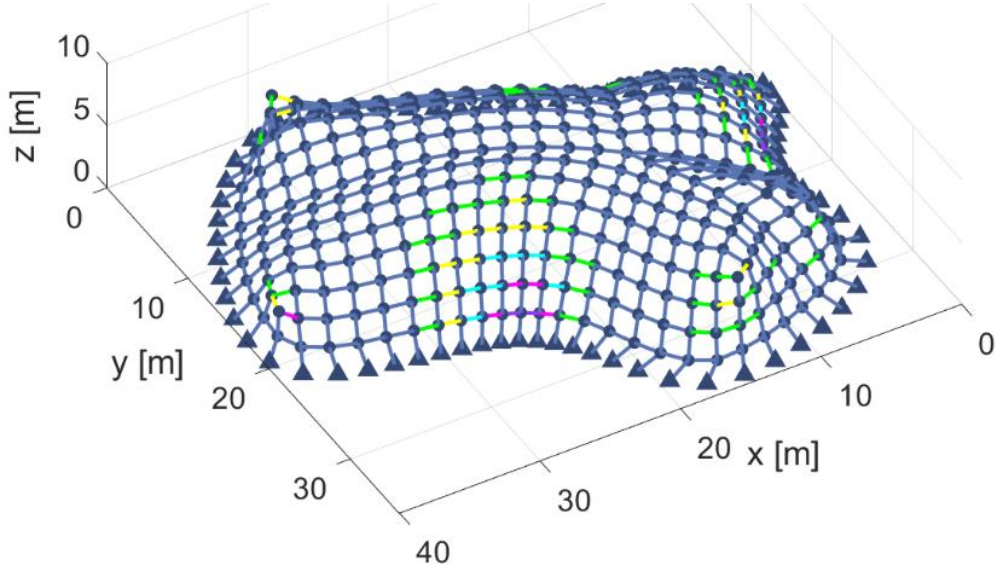
The i-MRA has two main advancements: - Multiple Orders MRA (**MO-MRA**)
- Repulsive Nodes MRA (**RN-MRA**)

**MRA**




**More than 100 different element typologies**

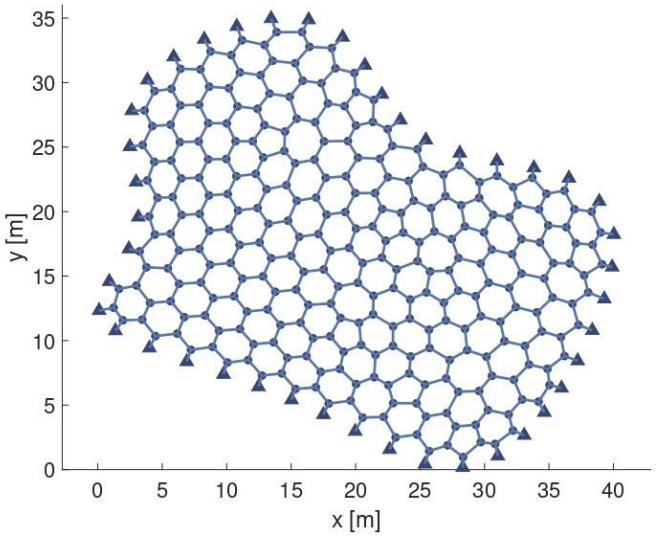
**i-MRA**



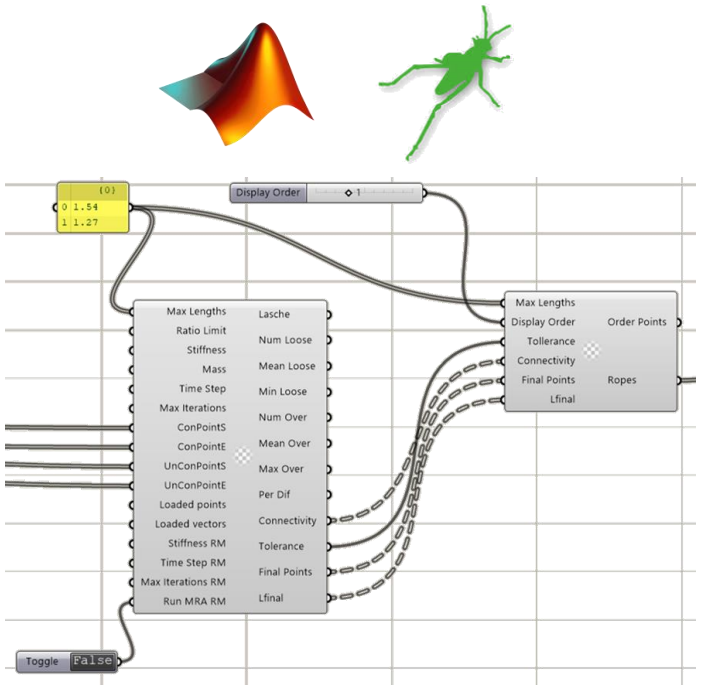

**Only 7 different element typologies**

# Improved Multibody Rope Approach (i-MRA)

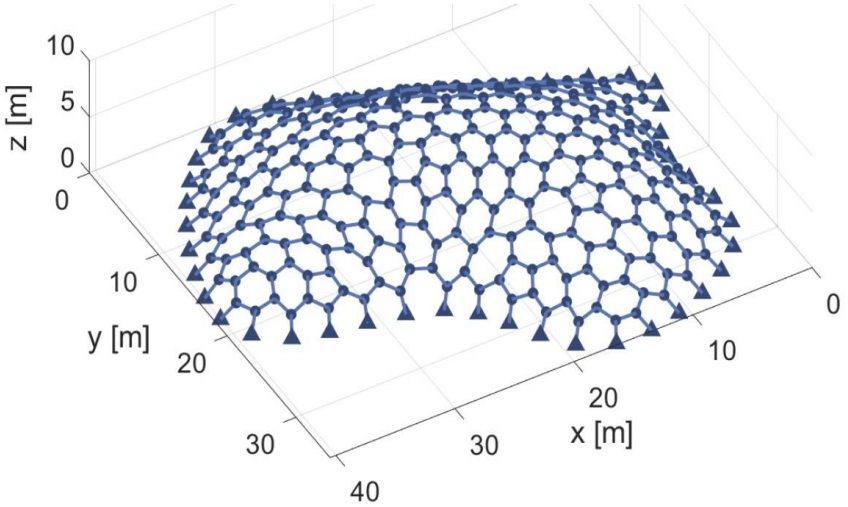
To facilitate practical implementation, a **parametric code** was developed using **MATLAB**, which was later converted to **C#** for integration with the parametric design software "**Grasshopper**".




**Basic mesh**



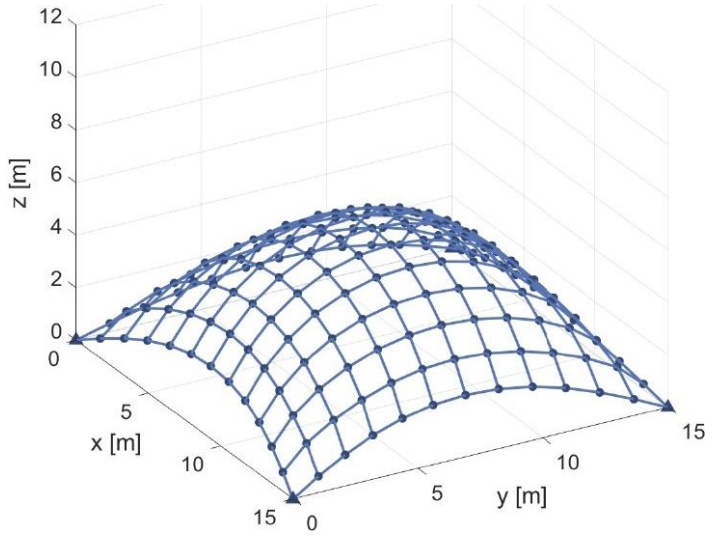

**i-MRA**



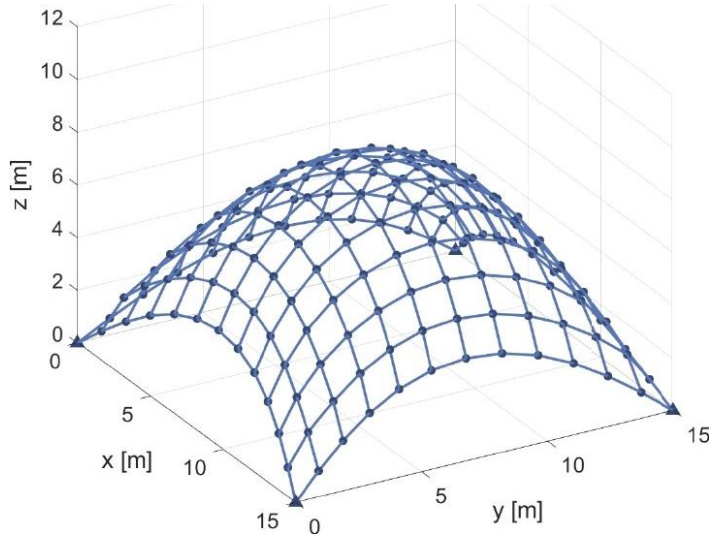

**Gridshell geometry**

The i-MRA is a **parametric method** in which the **main variables** that govern the final shape are the **slack coefficients ρ** assigned to the ropes:

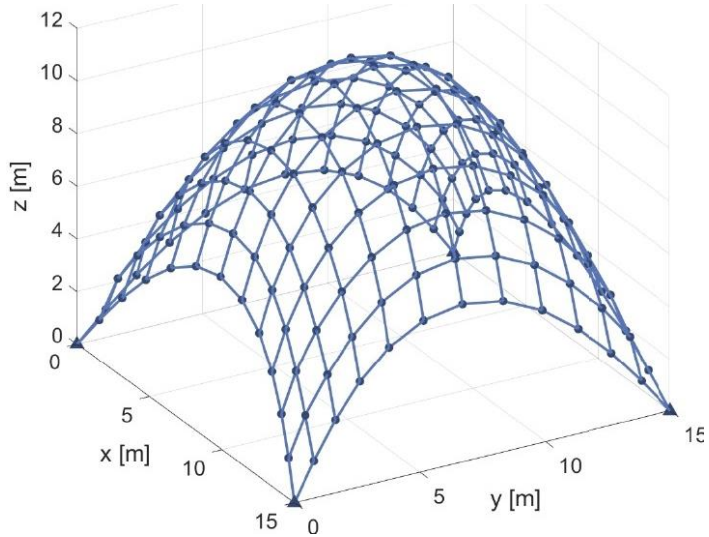
**ρ = 1.06**




**ρ = 1.12**




**ρ = 1.28**




## Which is the best geometry?

## STRUCTURAL OPTIMIZATION PROBLEM

# Multi-objective Size and Shape Structural Optimization Problem

**Structural optimization** seeks optimal values of **design variables** in order to maximize or minimize a specific quantity called the **objective function**, while satisfying a variety of conditions called **constraints**.

# Multi-objective Size and Shape Structural Optimization Problem

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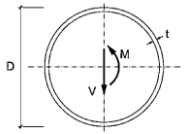
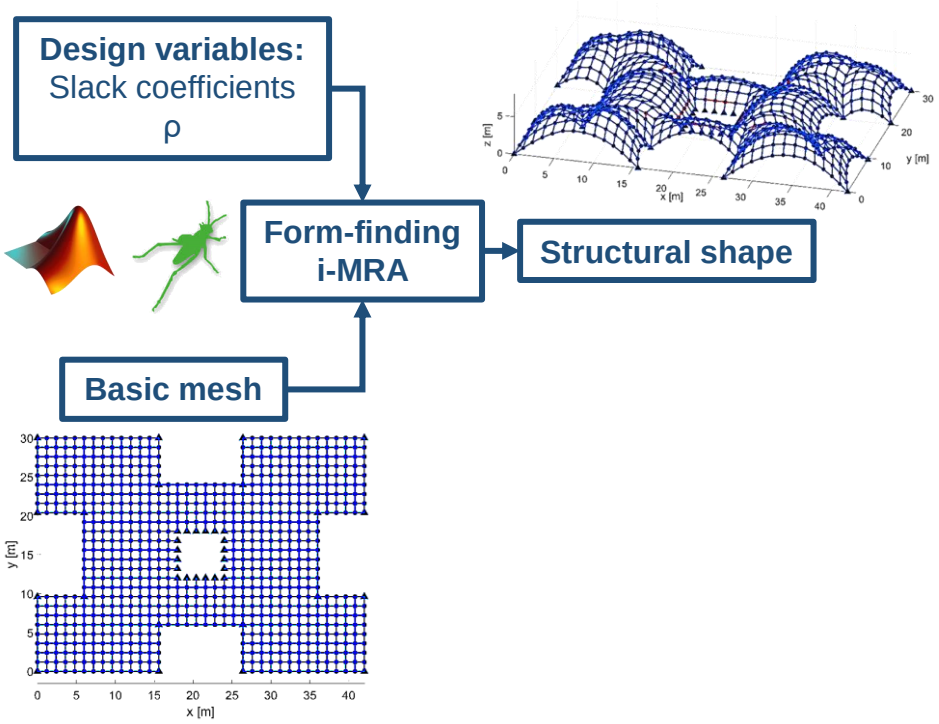
## Design Variables

- Slack coefficients: **ρ**

- Cross-section Area: **A**



# Optimization Workflow



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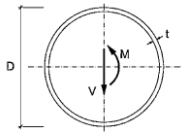
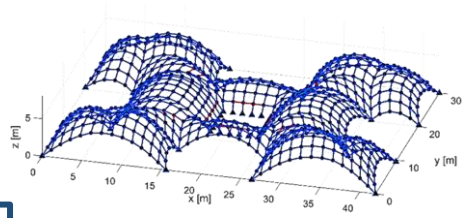


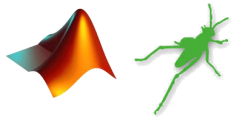

## Constraints

- Maximum Stress :
  $|\sigma_{max}| < f_y$
- Euler's buckling:
  $N < N_{cr}$
- Maximum dispacements:
  $\delta_{max} < \delta_{lim}$

# Optimization Workflow

**Design variables:**
Slack coefficients
ρ

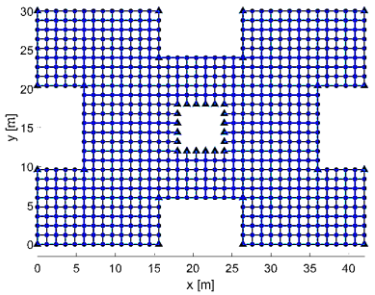




**Form-finding**
**i-MRA**

**Structural shape**

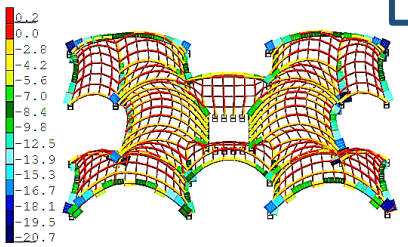
**Basic mesh**



**Finite Element Analysis**

**Internal actions**
**N, V, M**

**Structural verifications**
$\sigma < f_y$
$N < N_{cr}$
$\delta_{max} < \delta_{lim}$

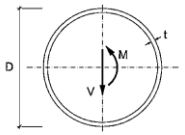


**Cross-section**
**A, J**

**Verified?**

YES

**Cross-section**
**Area**
**A**



**Increment cross-section properties**

NO

# Multi-objective Size and Shape Structural Optimization Problem

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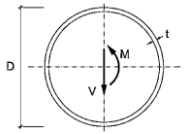
## Design Variables

- Slack coefficients: **ρ**

- Cross-section Area: **A**




## Constraints

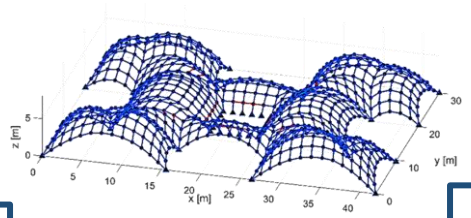
- Maximum Stress :
  $|\sigma_{max}| < f_y$
- Euler's buckling:
  $N < N_{cr}$
- Maximum dispacements:
  $\delta_{max} < \delta_{lim}$

## Objective functions

- Number of element types:
  $n_{type}$

# Optimization Workflow

**Design variables:**
Slack coefficients
ρ

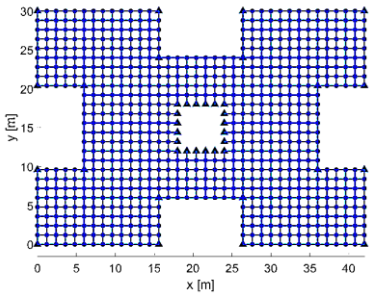


**Form-finding**
**i-MRA**

**Structural shape**

**Objective function 1:**
Number of element types
$n_{type}$

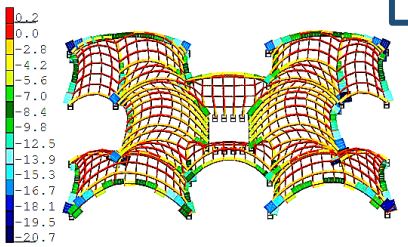
**Basic mesh**



**Finite Element Analysis**

**Internal actions**
**N, V, M**

**Structural verifications**
$\sigma < f_y$
$N < N_{cr}$
$\delta_{max} < \delta_{lim}$

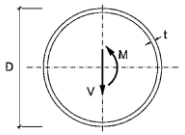


**Cross-section**
**A, J**

**Verified?**

YES

**Cross-section**
**Area**
**A**



**Increment cross-section properties**

NO

# Multi-objective Size and Shape Structural Optimization Problem

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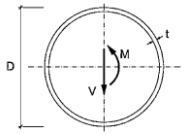
## Design Variables

- Slack coefficients: **ρ**

- Cross-section Area: **A**




## Constraints

- Maximum Stress :
  $|\sigma_{max}| < f_y$
- Euler's buckling:
  $N < N_{cr}$
- Maximum dispacements:
  $\delta_{max} < \delta_{lim}$

## Objective functions

- Number of element types:
  $n_{type}$
- Material consumption:
  $V$

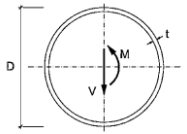
# Multi-objective Size and Shape Structural Optimization Problem

**Structural optimization** seeks optimal values of **design variables** in order to maximize or minimize a specific quantity called the **objective function**, while satisfying a variety of conditions called **constraints**.

## Design Variables

- Slack coefficients: **ρ**

- Cross-section Area: **A**




## Constraints

- Maximum Stress : $|\sigma_{max}| < f_y$
- Euler's buckling: $N < N_{cr}$
- Maximum dispacements: $\delta_{max} < \delta_{lim}$

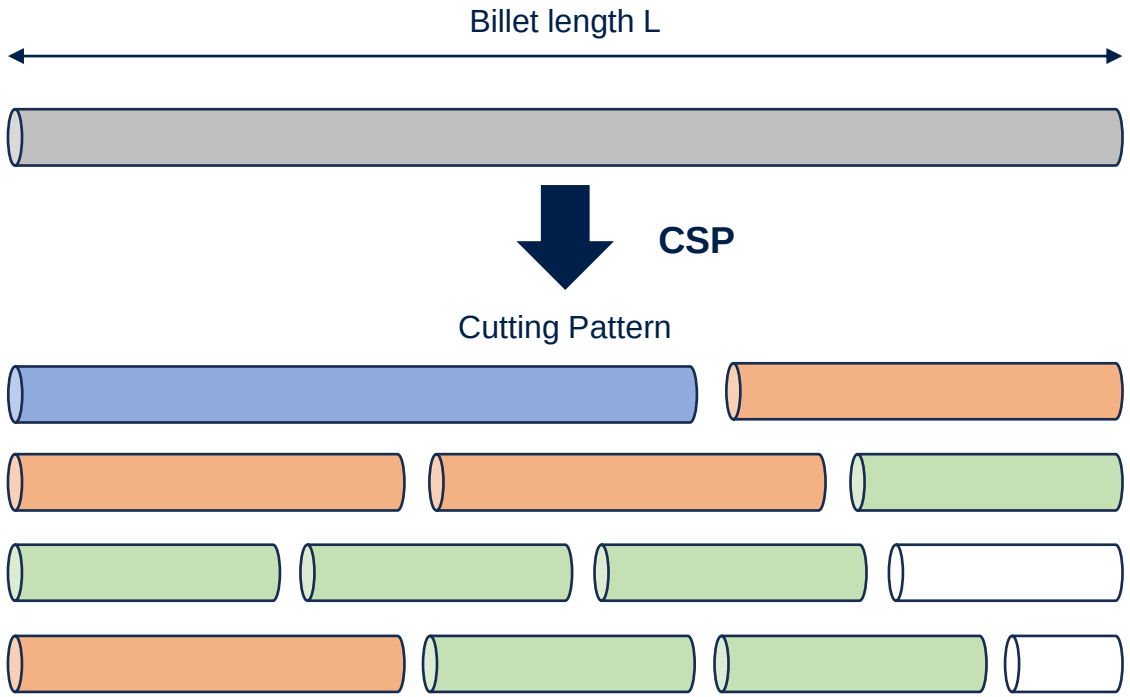
## Objective functions

- Number of element types: $n_{type}$
- Material consumption: **V**

**How to account for waste?!**

# Cutting Stock Problem (CSP)

The **CSP** is the problem of **cutting standard-sized pieces** of **stock material** into **pieces of specified sizes** while **minimizing material wasted**. It is an optimization problem that arises from applications in industry.



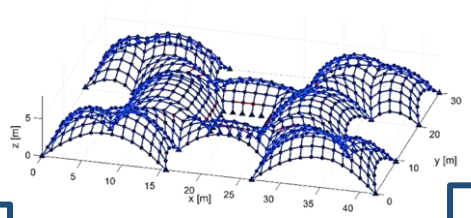

**Results**

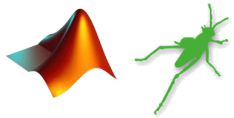
- **Material consumption** with waste
- **Cutting pattern** to minimize waste

Gilmore, P. C., & Gomory, R. E. (1963). A linear programming approach to the cutting stock problem—Part II. *Operations research*, *11*(6), 863-888.

# Optimization Workflow

**Design variables:**
Slack coefficients
ρ





**Form-finding**
**i-MRA**

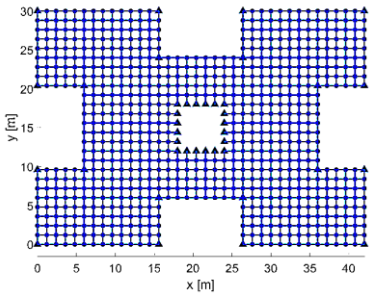
**Structural shape**

**Objective function 1:**
Number of element types
$n_{type}$

**Cutting Stock Problem**

**Number of billets**

**Basic mesh**



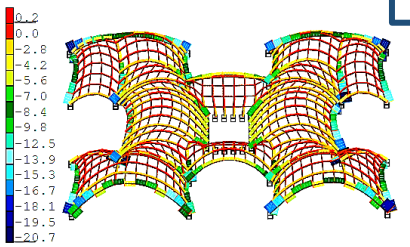
**Finite Element Analysis**

**Internal actions**
**N, V, M**

**Structural verifications**
$\sigma < f_y$
$N < N_{cr}$
$\delta_{max} < \delta_{lim}$

**Wastes**

**Objective function 2:**
Material consumption

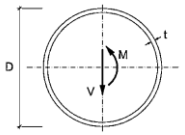


**Cross-section**
**A, J**

**Verified?**

YES

**Cross-section**
**Area**
**A**



**Increment cross-section properties**

NO

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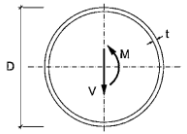
## Design Variables

- Slack coefficients: **ρ**

- Cross-section Area: **A**



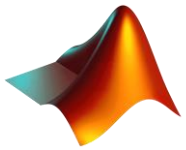

## Constraints

- Maximum Stress : $|\sigma_{max}| < f_y$
- Euler's buckling: $N < N_{cr}$
- Maximum dispacements: $\delta_{max} < \delta_{lim}$

## Objective functions

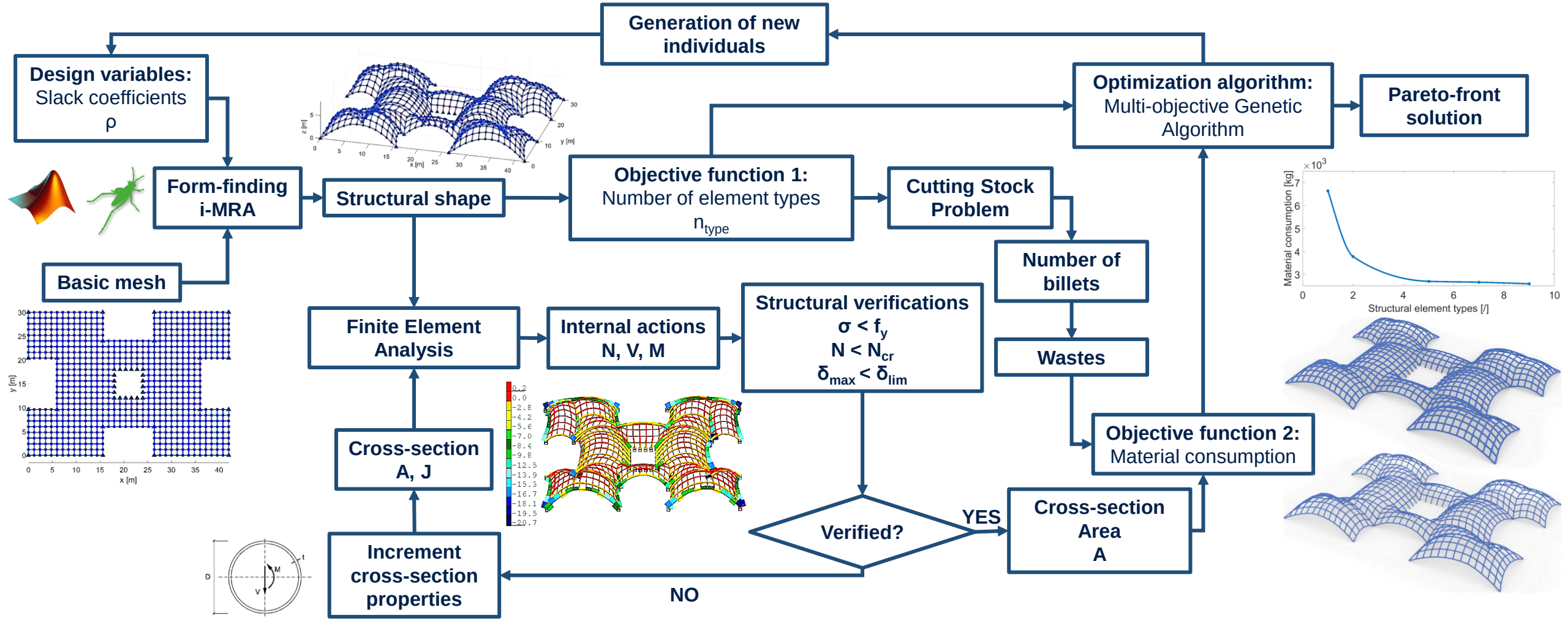
- Number of element types: $n_{type}$
- Material consumption: $V$

## Optimization algorithm



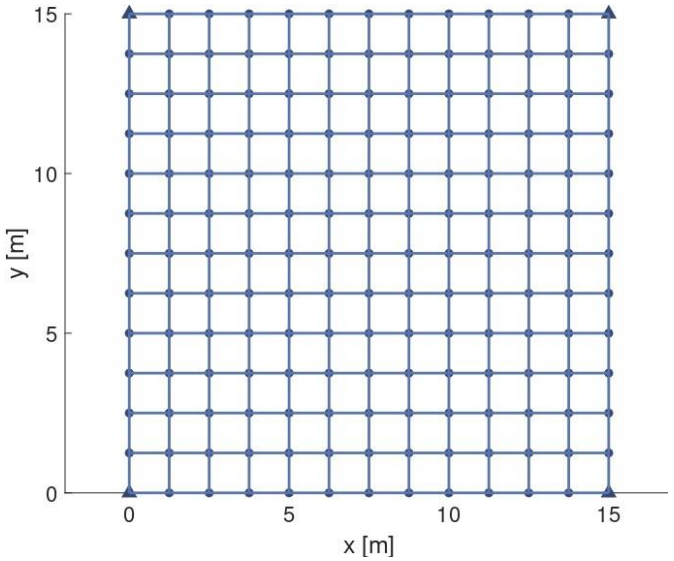
Non-dominated sorting multi-objective Genetic Algorithm

# Optimization Workflow



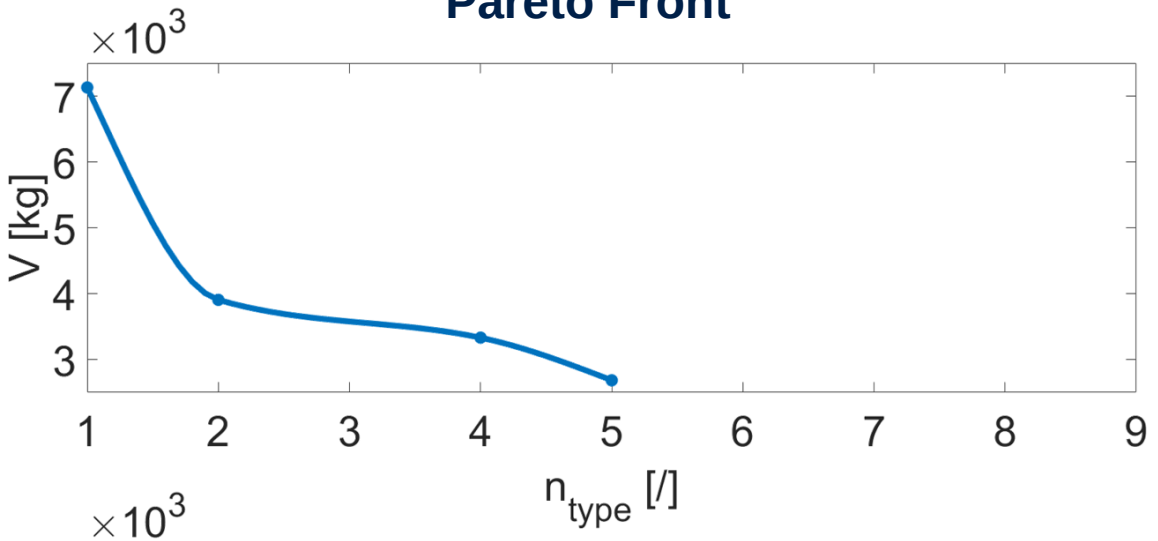
# Optimization results

## Base mesh

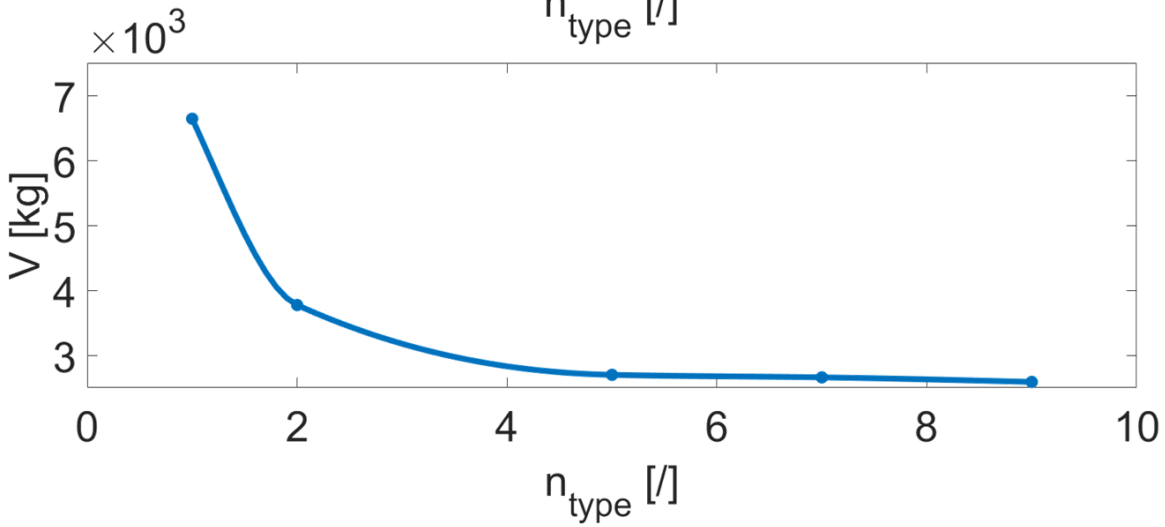



## Pareto Front

**CSP**




**No CSP**



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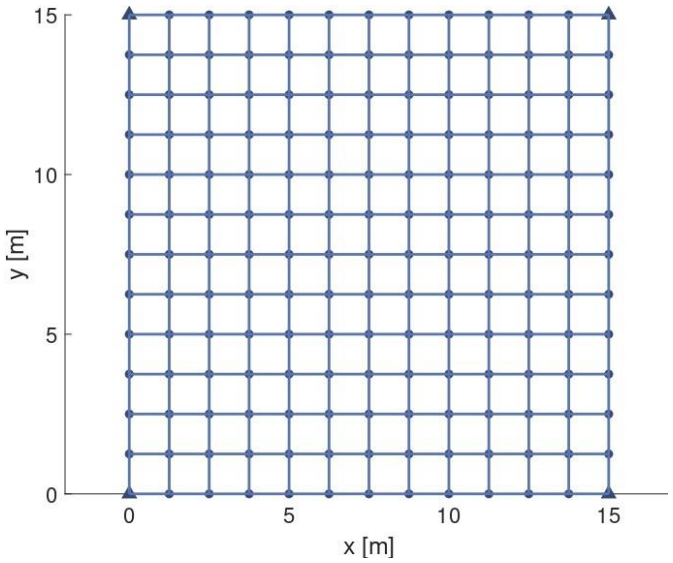
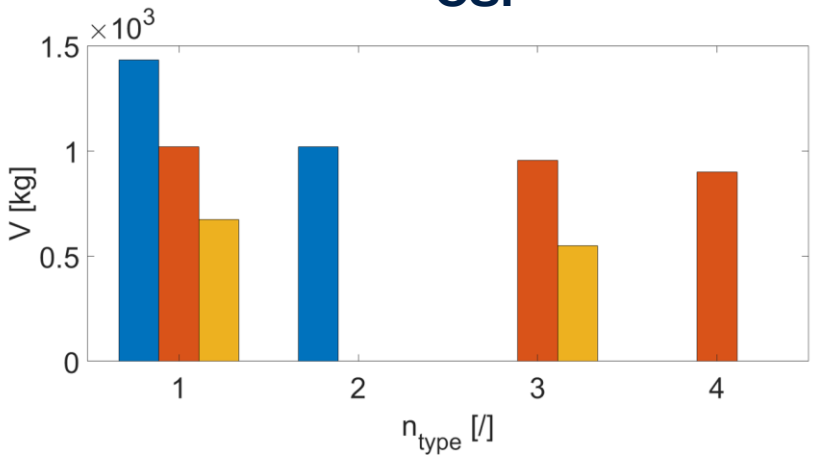
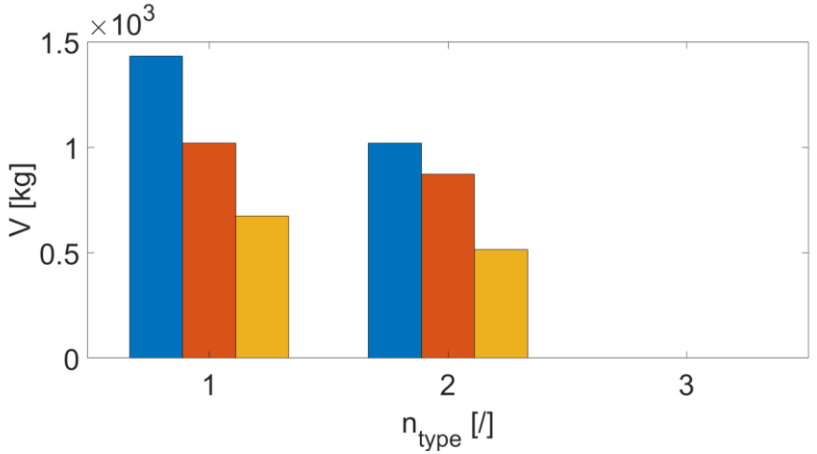
## Base mesh

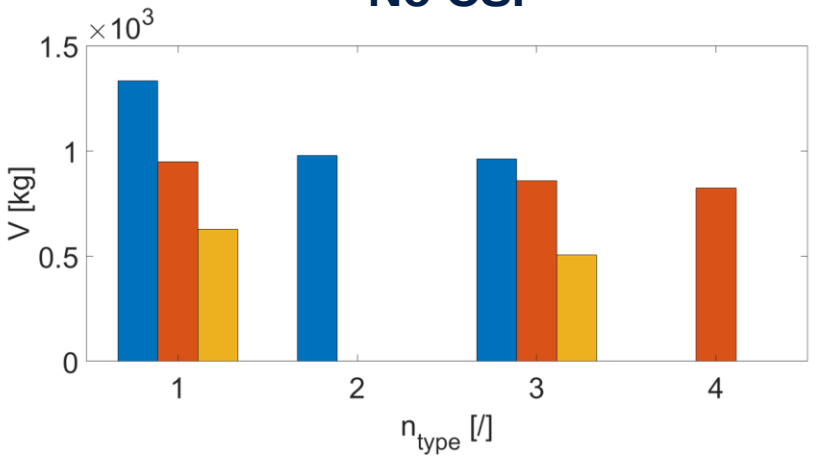


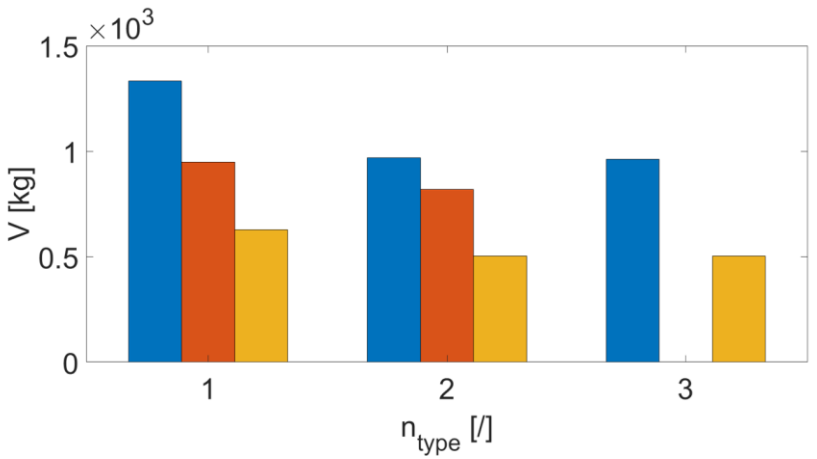

## CSP




## No CSP

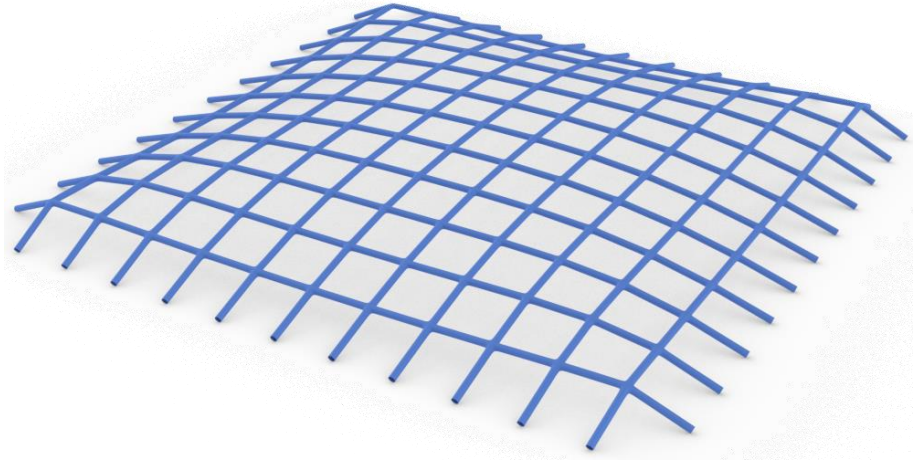


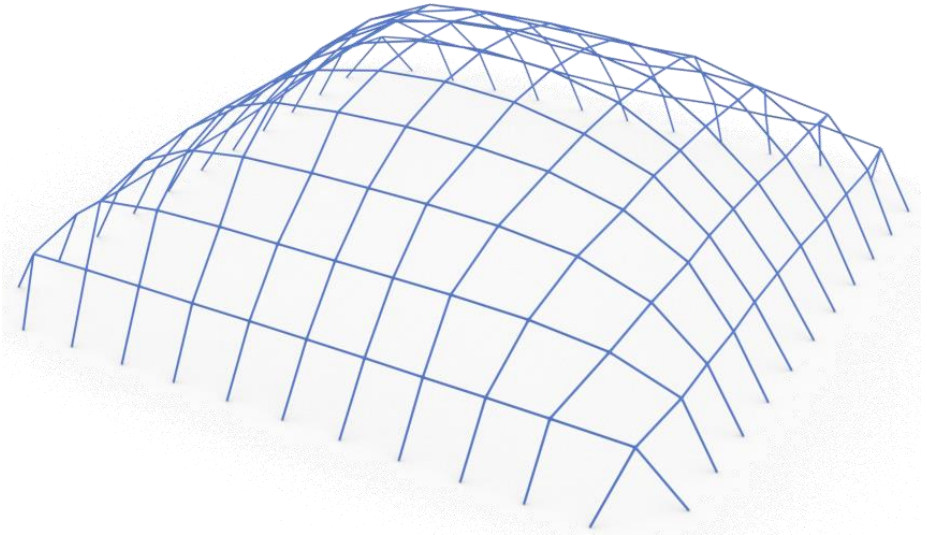

1 var

2 var

# Optimal configurations



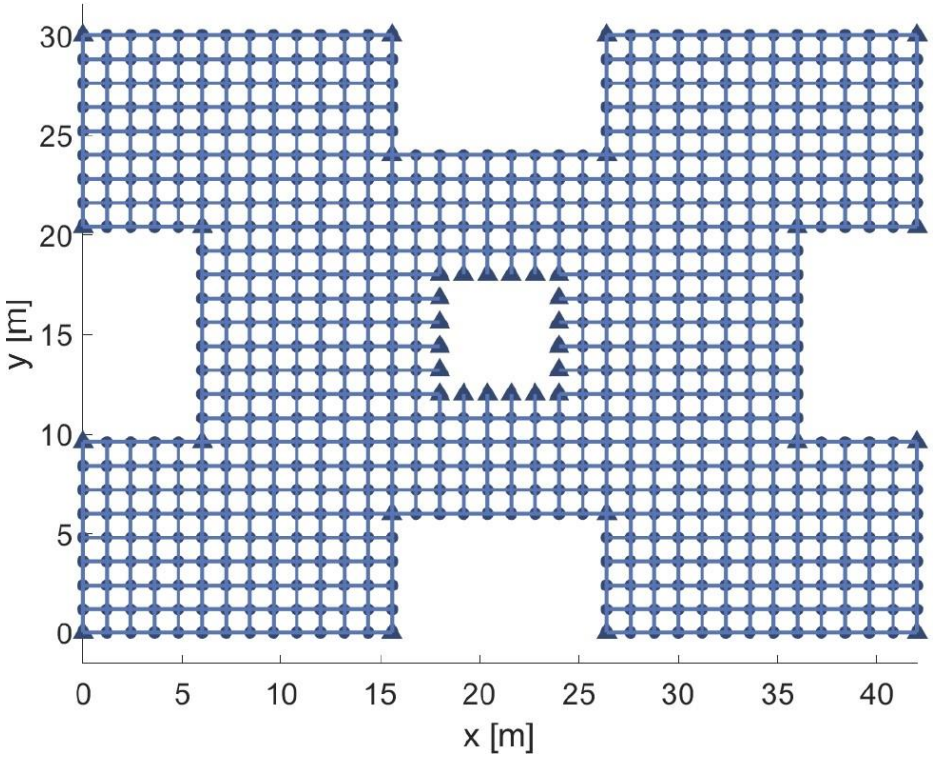
$n_{type} = 1$ V = 7.13 t



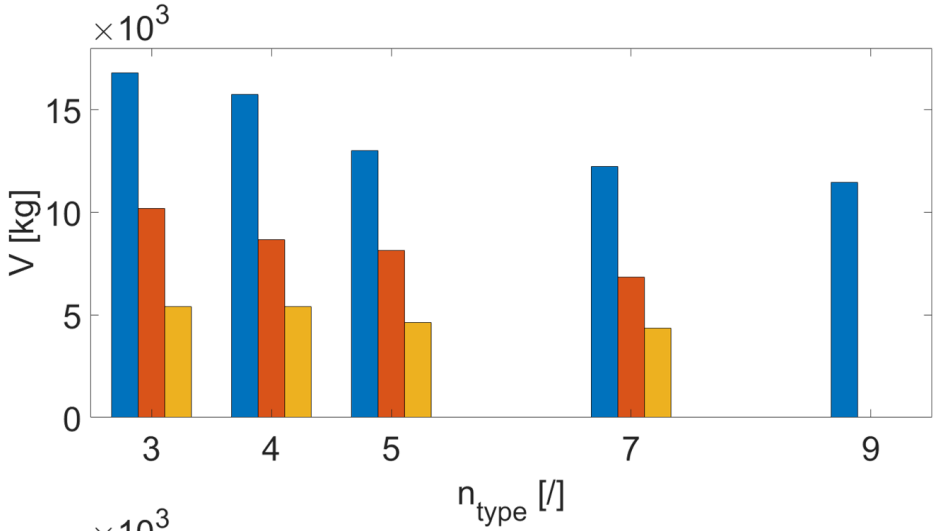
$n_{type} = 2$ V = 2.55 t

# Optimization results

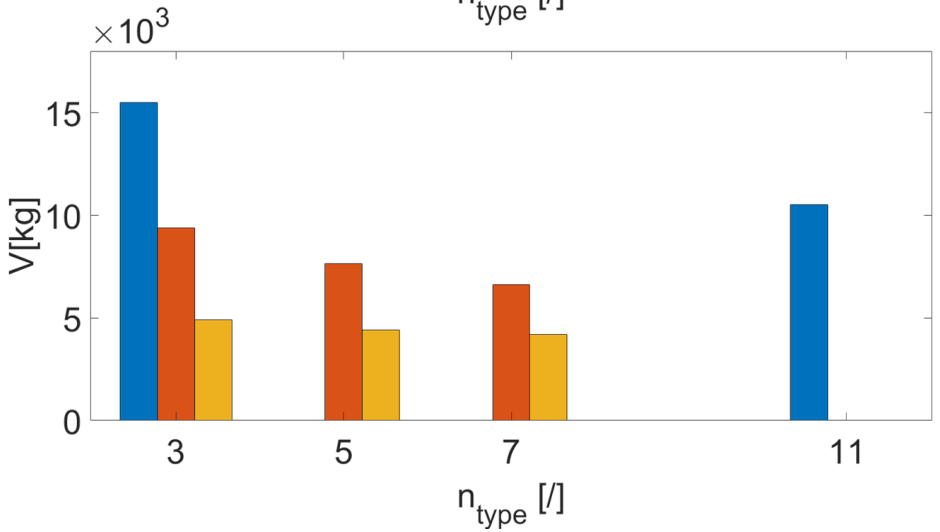
## Base mesh



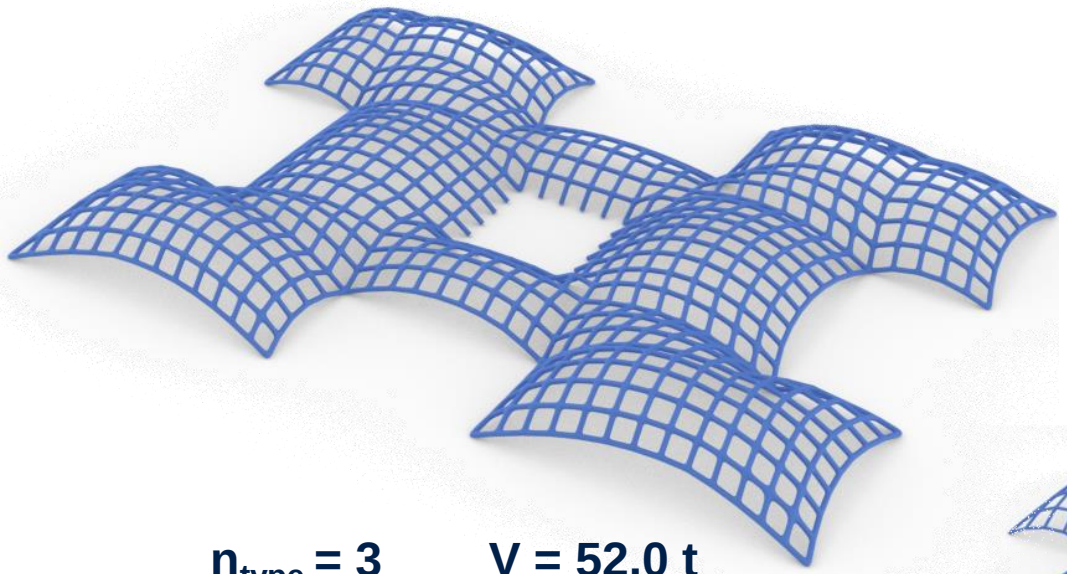

**CSP**



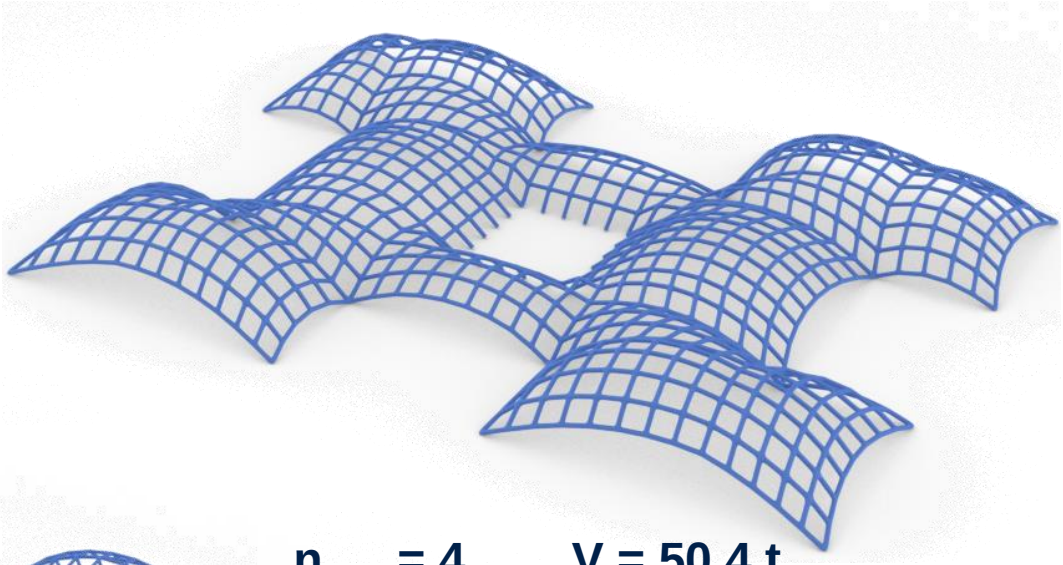

**No CSP**



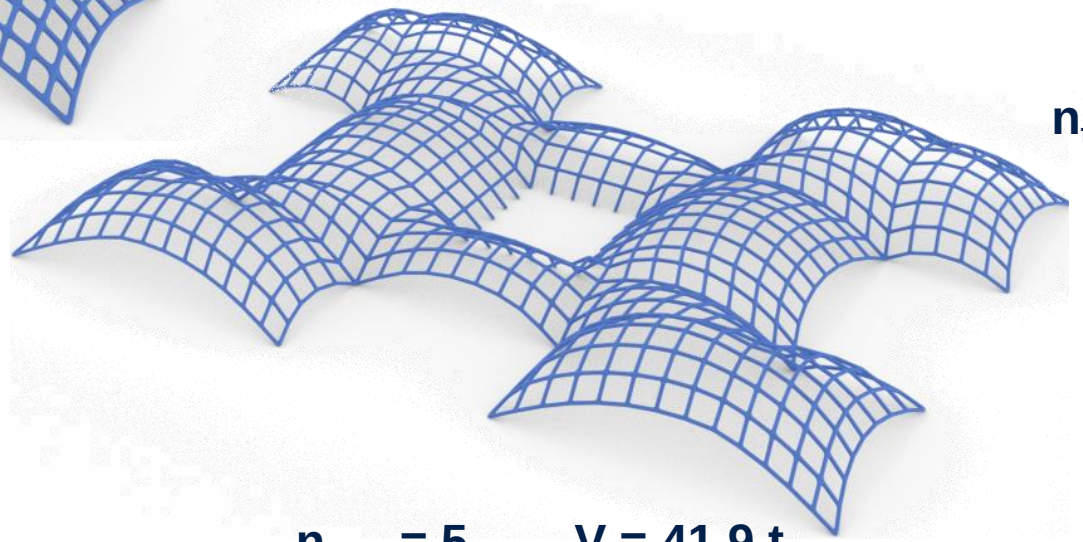
# Optimal configurations



$n_{type} = 3$ V = 52.0 t



$n_{type} = 4$ V = 50.4 t



$n_{type} = 5$ V = 41.9 t

# Conclusion

In this work:

- A new method (**i-MRA**) for **form-finding** of **free-form gridshell structures.**
- The i-MRA perform **form-finding** while **reducing the construction complexity** by using standard piece.
- **Matlab code** to solve the new form-finding procedure.
- A user-friendly **Grasshopper component** designed to enable practitioners to easily utilize the i-MRA.
- A **multi-objective optimization procedure combined with the form-finding** process of gridshell structures.
- Procedure to **minimize material consumption** and **reduce complexity** during the construction phase.
- Use of the **Cutting Stock Problem** to **minimize waste** and define the best industrial cutting **patterns**.
- **Application examples** showcasing the results.

# THANKS FOR THE ATTENTION

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